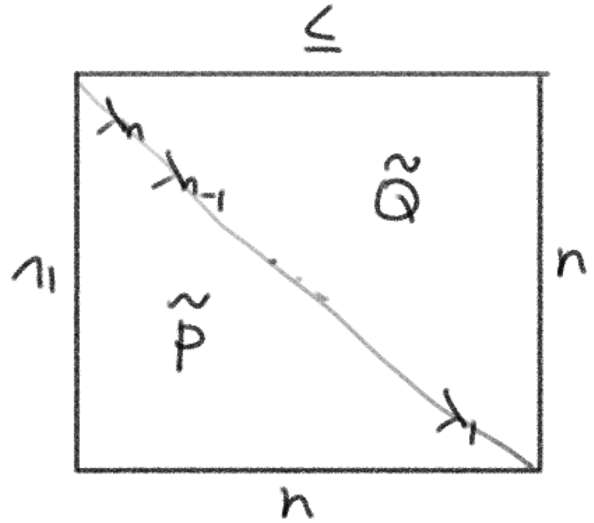
In RSK, we have a pair of SSYT (P, Q)

we can turn them into GT patterns

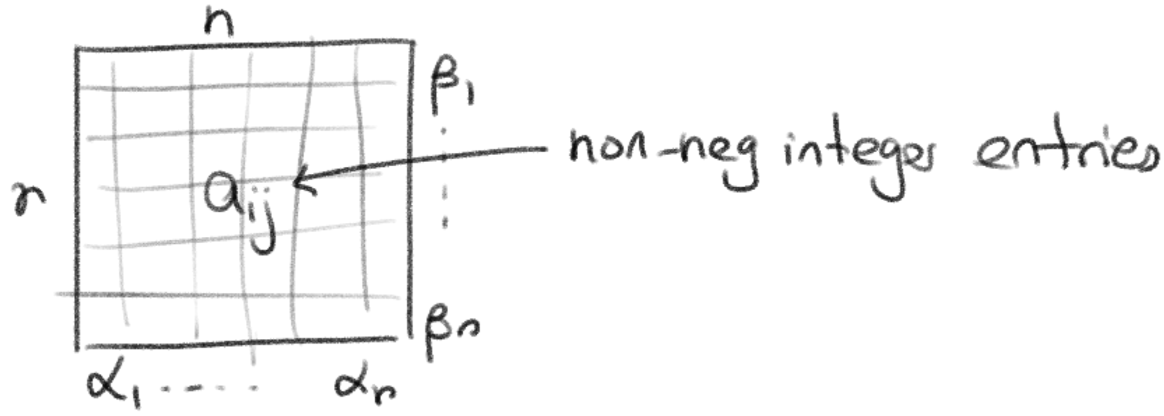
$\tilde{P}$ and $\tilde{Q}$

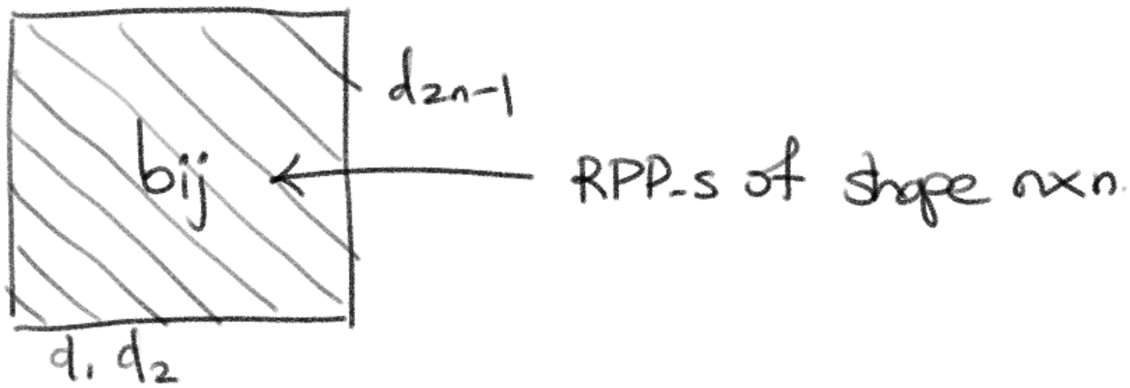
glue them together into a square



we get reverse plane partitions (RPP)

So we get correspondence





$$d_1 = \alpha_1 \qquad d_{2n-1} = \beta_1$$
$$d_2 = \alpha_1 + \alpha_2 \qquad d_{2n-2} = \beta_1 + \beta_2$$
$$\vdots \qquad \vdots$$
$$d_n = \alpha_1 + \cdots + \alpha_n \qquad d_n = \beta_1 + \cdots + \beta_n$$

This shows that RSK is consistent with transposition

Similar bijection works for any Young diagram.

A generalization of RSK

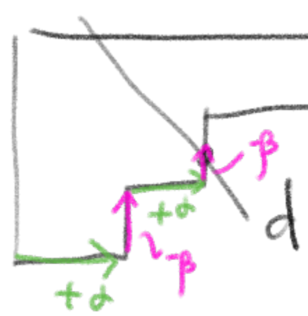
Let $\mathcal{K}$ be a Young diagram.

<u>Thm</u> There exists a bijection $\varphi_{\mathcal{K}}$

$$\left\{\begin{matrix}\text{non-neg matrices}\\ \text{of shape } \mathcal{K}\end{matrix}\right\} \xrightarrow{\varphi_{\mathcal{K}}} \left\{\begin{matrix}\text{RPP-s of}\\ \text{shape } \mathcal{K}\end{matrix}\right\}$$

such that

- $\varphi_{\mathcal{K}}$ is piecewise linear map.
- colsums $\alpha_1, \dots, \alpha_n$ and rowsums $\beta_1, \dots, \beta_m$ of A are related to diagonal sums $d_1, \dots, d_{m+n-1}$ of $B = \varphi_{\mathcal{K}}(A)$



- symmetry: if $\varphi_K : A \to B$

  then $\varphi_{K'} : B \to A$

- If $K = n \times n$ agrees with RSK

when $\alpha_1 = \dots = \alpha_n = \beta_1 = \dots = \beta_n$ the bijections specialize into a bijection between rook placements and oscillating tableus